Physics of the Flat-Top Jump

When one rides a motorcycle slowly off a flat top jump, the front of the motorcycle drops quickly and the rider often orbits over the bars! When the jump is ridden quickly and aggressively, however, the motorcycle lands flat on both wheels (and you look good doing it). What accounts for this difference?

Video (by Jesse Ziegler): Ryan Dudek pops a nice one off a flat-top boulder.
http://www.motorcyclejazz.com/VIDEOS/P9220400.MP4

When a motorcycle is on flat ground, gravity exerts a force on the *center of mass* of the motorcycle that is balanced by normal forces exerted by the ground on the motorcycle at the front and rear wheels. The first and second conditions of equilibrium are met. $\Sigma F = 0, \Sigma\Gamma = 0$

As the front wheel of the motorcycle leaves the jump the normal force on it disappears. Notice that the force of gravity acting on the c.m. of the motorcycle begins to exert a *torque* about the rear axle of the motorcycle. This torque produces an *angular acceleration* that, in turn, produces an *angular displacement* which increases with time, i.e., the front end of the motorcycle begin to drop.

When the rear wheel clears jump the normal force on it also disappears. The torque produced by the force of gravity acting on the c.m. about the rear axle disappears and the second condition of equilibrium is now met, once again. The motorcycle does, however, continue to rotate about the rear axle, but now with constant angular speed. If the time interval over which the angular acceleration occurs is large, this angular velocity is much greater than if the interval is small. The force of gravity acting on the c.m. continues to accelerate the entire bike toward the ground while the front end continues to drop.

The key to understanding how it's possible for the motorcycle to land on both wheels at the same time under these circumstances lies in understanding that when the motorcycle leaves jump horizontally at high speed, the time interval between the normal force disappearing on the front and rear wheels, the time interval in which the force of gravity exerts a torque about the rear axle, is very short (at most, a few tenths of a second). During this short time the torque produced by gravity about the rear axle produces only a small angular acceleration resulting in a small angular velocity and small angular displacement. If the journey to the ground is similarly short, this produces a very small (indiscernible) change in the attitude of the motorcycle. The front wheel simply doesn't drop far enough in the allotted time interval to be discernable.

If the horizontal speed is low, the time interval over which the gravitational force on the c.m. applies a torque about the rear axle is large, resulting in a larger angular acceleration and a larger angular rotation about the rear axle than before – the front wheel may hit the ground before the rear wheel even leaves the top of the jump.

If the distance to the ground is large, even a small angular velocity about the rear axle may result in a large angular rotation. In this case it's necessary to do something to counter the effect of this rotation. Conservation of angular momentum is the key to accomplishing this.

In the accompanying video, the horizontal speed of the motorcycle (ridden by Ryan Dudek, of Cycle World) as it leaves the rock is not great enough to prevent the front end from dropping a lot more than it does. Conservation of angular momentum is responsible. Note that when the front end of the motorcycle drops, angular momentum is not conserved because the second condition of equilibrium is not met ($\Sigma\Gamma \neq 0$) during the interval in which the front and rear wheels leave the jump. The rider compensates for this by accelerating the rear wheel using a torque supplied by the engine. As the rear wheel increases its rate of forward rotation about the rear axle, the rest of the motorcycle (and rider) rotate in the opposite direction about the same axis as the system attempts to conserve angular momentum.

The same general concept applies in a variety of motorcycle jumps and wheelies.

Let's say that you were attempting to jump a large ditch on your motorcycle. The ditch is about 3 meters wide, 2 meters deep and both edges of the ditch are at the same vertical level. Your motorcycle has 0.3 meters of suspension travel. The mass the motorcycle is 115 kg and your mass is 70 kg. How fast do you have to be going to be able to clear the ditch with a minimal jolt?

Since both sides of the ditch are at the same height, the motorcycle will experience a vertical displacement that is guaranteed to place it below the opposite side of the ditch while in the air. If this displacement is small, you have a chance of clearing the opposite side of the ditch with a minimal jolt – as long as the vertical displacement of the bike is within the range of the suspension travel and the attitude of the motorcycle is level, or, better yet, front wheel up.

Let's assume that a motorcycle has suspension, front and rear, with 0.3 meters of travel. As long as the bike doesn't drop more than 0.3 meters in 3 meters flight across the ditch, the suspension has a chance of soaking up the impact due to the drop.

$$y - y_0 = \frac{1}{2}at^2 \rightarrow \sqrt{\frac{2(-0.3m)}{-9.8m/s^2}} = t = 0.25s$$

This means that you have to cover 3 meters in about a quarter of a second to stay within the range of the motorcycle's suspension travel.

$$x - x_0 = v_{0x}t \rightarrow \frac{3m}{0.25s} = t = 12m/s$$

This is a little less than 30 mph - clearly an attainable speed.

Now let's look at the angular acceleration of the motorcycle around the rear axle that occurs during the interval of time between the front and rear wheels leaving the edge of the ditch. Assume that the distance between the front and rear wheel contact patches is about 1.5 meters. The motorcycle then spends, at a translational speed of 12 *m/s*, about 0.125 seconds undergoing an angular acceleration about the rear axle. The torque produced by the force of gravity about the rear axle is a combination of the torque produced by the mass of the bike about the rear axle and the torque produced by the mass of the rider. Since the rider generally is positioned well aft of the motorcycle's c.m.

$$\Sigma\Gamma = (M_{bike}g)\left(\frac{r}{2}\right)sin\,\theta + m_{rider}\,g\,\frac{r}{4}\,sin\,\theta$$

$$(115kg)(9.8m/s^2)\left(\frac{1.5m}{2}\right)(1) + (70kg)(9.8m/s^2)\left(\frac{1.5m}{4}\right)(1) = 1102N\cdot m$$

Let's assume that the moment of inertia of a motorcycle about the rear axle is approximately the same as the moment of inertia of a rod hinged about an end. Let's then use the length of the bike between the wheels as its approximate length.

$$I = \frac{1}{3}M_{bike}r^2 = \frac{1}{3}(115kg)(1.5m)^2 = 86kg\cdot m^2$$

The moment of inertia of the rider about the rear axle at a distance of 0.75 meters from the rear axle is $m_{rider}r^2$ or:

$$I_{rider} = m_{rider}r^2 = (70kg)(0.38m)^2 = 10kg\cdot m^2\,.$$

So the total moment of inertia of the motorcycle and rider is:

$$I_{total} = I_{motorcycle} + I_{rider} = 96kg\cdot m^2$$

Now, using Newton's second law for rotation $\Gamma = I\alpha$ we may determine the approximate angular acceleration of the motorcycle about the rear axle:

$$\Gamma = I\alpha \rightarrow \frac{\Gamma}{I} = \alpha = \frac{1102N\cdot m}{96kg\cdot m^2} = 11.5rad/s^2$$

Using rotational kinematics, we can compute the approximate drop of the front end of the motorcycle. The time that the motorcycle undergoes acceleration about the rear axle is 1.25 seconds, so at an angular acceleration of 11.5 rad/s^2, the angular speed is:

$$\omega = \omega_0 + \alpha t \rightarrow \omega = (11.5rad/s^2)(0.125s) = 1.4rad/s$$

This produces an angular displacement, during the acceleration phase, of:

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \rightarrow \theta - \theta_0 = \frac{1}{2}(11.5rad/s^2)(0.125)^2 = 0.09rad$$

After the rear wheel leaves the ditch, the rotational acceleration stops but the bike continues to rotate at a constant speed:

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \rightarrow \theta - \theta_0 = (1.4 rad/s)(0.125) = 0.18 rad$$

for a total rotation of about 0.3 radians. Since 1 radian is about 57.3 degrees, this is a rotation of about 17 degrees. The front wheel of the bike will drop about 0.4 meters, if unchecked. This is beyond the 0.3 meter limit of the suspension.

In order to keep the front end of the motorcycle level, we need to counter the torque supplied by gravity acting on the rider and c.m. about the rear axle with an equal but opposite torque. This is accomplished by accelerating the rear wheel with a torque supplied by the engine. If these torques balance, the second condition of equilibrium is met and the front end does not drop (or drops a lot less if they are close).

The engines of most dirt bikes are capable of generating about 45 $N \cdot m$ of torque at the crankshaft. To convert this to torque about the rear axle we have to take into account the effect of the drive train: transmission, chain and sprockets – all of which serve as torque multipliers. To do this we multiply the primary ratio (crankshaft to clutch) by the drive gear ratio (transmission gears) by the final gear ratio (front and rear sprockets). Typical values for a bike in a low gear would be:

$$(2.739)(2.231)(3.923) = 24.$$

This means that the engine torque is multiplied by a factor of 24. In this case the engine generates about 1100 N · m of torque at the rear wheel. So it *is* possible to counter the torque produced by gravity with a torque from the engine. If you produce this counter torque for the whole 0.25 seconds the motorcycle is in the air, it is possible to actually raise the front end.